Optical Spanners (GR/K61838)

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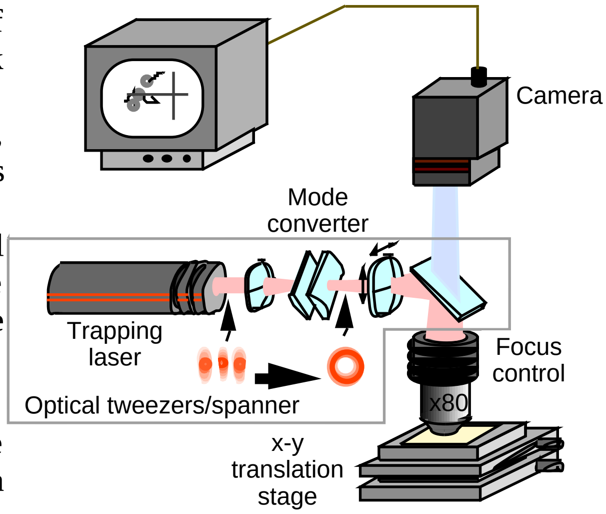
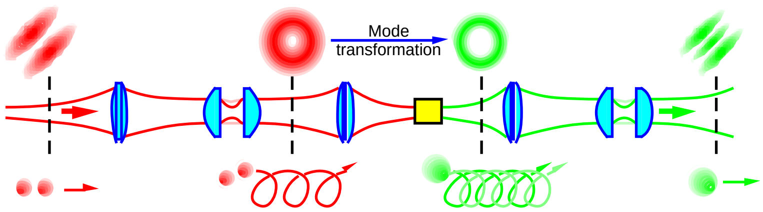


Initially this grant funded the completion of the work on the *optical spanners*.[1]. This work has received widespread public exposure including "Science Now", BBC Radio 4, "Science" and the "Financial Times". This aspect of the work was extended and specifically led to evidence that an optical tweezers using a Laguerre-Gaussian mode results in an improved system, halving the required laser power.

Additionally the "blue-sky" nature of the ROPA award has enabled us to open a number of new research areas:

The orbital angular momentum of a Laguerre-Gaussian modes is $\ell\hbar$ per photon. Consequently, the longer the wavelength of the beam the higher the orbital angular momentum per photon. With this grant we extended the work to generate *Laguerre-Gaussian modes at mm-wave frequencies*[2]. We have now used these mm-wave beam in conjunction with a rotating Dove prism to perform the first observation of the *Rotational Frequency Shift*[3] which we also interpret in terms of earlier work. These beams have also found application within the latest generation of military mm-wave antenna



Within this grant we considered for the first time the role that orbital angular momentum plays in non-linear optics. *Second harmonic generation using Laguerre-Gaussian modes*[4]. The orbital angular momentum represents an additional constraint in the phase matching process which leads to a mode transformation. This work opened a new research field which has received further funding from the EPSRC.

Other "blue-sky" research funded under this grant includes the identification and generation of a class of astigmatic beams which contain orbital angular momentum of many 100's of $\hbar$ per photon[5] and a new method for the generation of Laguerre-Gaussian laser modes using a stressed fibre optic waveguide[6].

Key Publications

1. Opt. Lett. **22**, 52-54 (1997), N B Simpson, K Dholakia, L Allen and M J Padgett
2. Opt. Commun. **127**, 183-188 (1996), G A Turnbull, D A Robertson, G M Smith, L Allen and M J Padgett
3. Submitted to Phys. Rev. Lett. (November 1997), J Courtial, K Dholakia, L Allen and M J Padgett
4. Phys. Rev. A, **54**, R3742-R3745 (1996), K Dholakia, N B Simpson, M J Padgett and L Allen
5. Opt. Commun. **144**, 210-213, J Courtial, K Dholakia, L Allen and M J Padgett
6. Accepted for publication in Appl. Opt. (July 1997), D McGloin, N B Simpson and M J Padgett

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Summary

The research funded by this grant concerns the use of Laguerre-Gaussian modes and the development of an optical spanner and improved optical tweezers. Additionally, the "blue-sky" nature of the ROPA funding has enabled us to open several new research areas related to Laguerre-Gaussian laser modes and the orbital angular momentum of light:

- completion of the development of the optical spanner
- verification of the improved optical tweezers based on a Laguerre-Gaussian modes
- the first study of role of orbital angular momentum in frequency doubling
- identification and generation of beams with very high orbital angular momentum
- new technique for generation of Laguerre-Gaussian modes
- first observation of the recently predicted Rotational Frequency Shift

Introduction

It is well known that light can have an angular momentum associated with it. Circularly polarised light carries an angular momentum, that relates to the intrinsic spin of individual photons. This was first demonstrated by Beth in 1936[1] who experimentally observed the torque exerted on a birefringent plate as the polarisation state of the transmitted light was changed from *LH* circularly-polarised to *RH* circularly-polarised.

It was less well known was that a light beam could also have orbital angular momentum associated with it. In 1992 Allen *et. al.*[2] predicted that for Laguerre-Gaussian (*LG*) modes the orbital angular momentum is well defined and has a value of $\ell\hbar$ per photon, where ℓ is the azimuthal index of the mode. The presence of this orbital angular momentum can be deduced classically from Maxwell's equations. It originates from the azimuthal dependence of the phase of a *LG* mode, which has the form, $\exp(i\ell\phi)$. Although the magnitude of this orbital angular momentum is quantised in terms of the number of photons in the beam, it should be emphasised that as with polarisation, this is not an exclusively quantum property. It is a gross property of the beam which can be understood classically in terms of the electric and magnetic fields.

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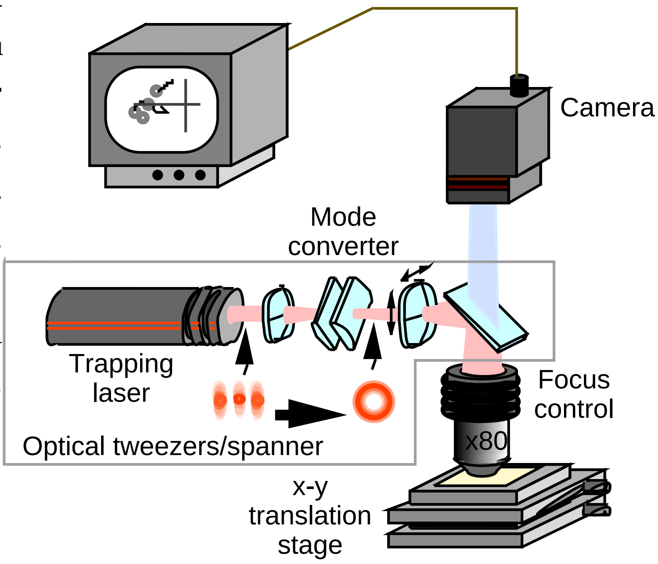
[1] R A Beth, Phys. Rev. **50**, 115 (1936)

[2] L Allen, M W Beijersbergen, R J C Spreeuw and J P Woerdman, Phys Rev. A, **45**, 8185 (1992)

Improved Optical Tweezers

Since the early 1970's Ashkin has pioneered techniques for the optical trapping of small particles by laser beams[3]. A single beam of tightly focussed laser light creates an extremely high electric field gradient in the vicinity of the focus. This is similar to the force which draws a dielectric into the high field region of a capacitor; a dielectric particle falling within the laser beam will experience a force which is directed towards the focus of the beam. In later work Ashkin demonstrated that, providing the numerical aperture of the focusing optics is high, the optical tweezers give rise to a 3-dimensional trap[4]. The force due to the field gradient is sufficiently high to overcome the forces due to both gravity and radiation pressure. Such traps are commonly referred to as optical tweezers and are now widely used in biological applications.



We demonstrated that the *LG* mode produces a stronger tweezers force than the fundamental Gaussian mode used previously. For a wide range of particle sizes the laser power required with an $\ell = 3$ Laguerre-Gaussian mode is reduced by a factor of two [5]. This is important for biological applications where reduction of the laser power lessens the risk of damage to the living sample.

We are now working with the CNRS in Grenoble to investigate the use of these improved optical tweezers to position magnetic samples within miniature electromagnets.

Optical Spanners (in conjunction with GR/K11536)

In relation to the angular momentum content of Laguerre-Gaussian modes there was one key question. Could the orbital angular momentum be transferred to a material object? Previous attempts to demonstrate this transfer had been based on suspended cylindrical lenses and had failed due to difficulties in alignment.

One thrust of our work has been to use a *LG* beam within an optical tweezers and make a mechanical measurement of the orbital angular momentum inherent in a *LG* laser mode as it is partially absorbed by the trapped particle. By using the same beam to trap the particle on the beam axis, and therefore defining the rotation axis, we overcame the problems associated with mis-alignment.

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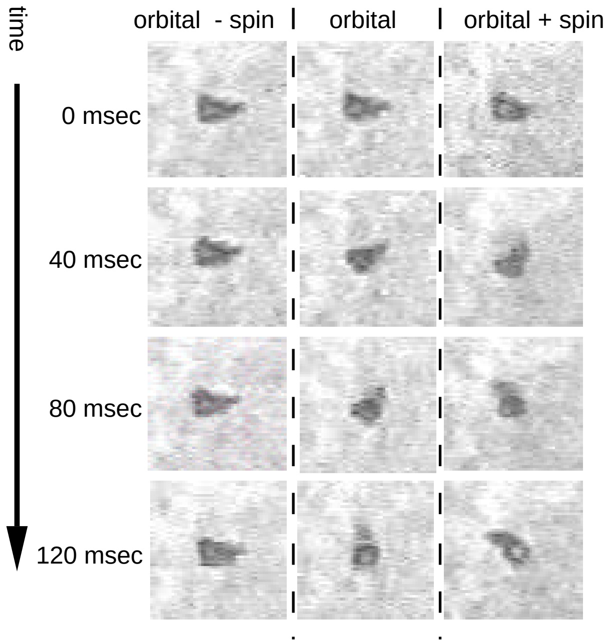
[3] A Ashkin and J M Dziedzic, App. Phys. Lett., **19**, 283 (1971)

[4] A Ashkin, J M Dziedzic, J E Bjorkholm and S Chu, Opt. Lett., **11**, 288 (1986)

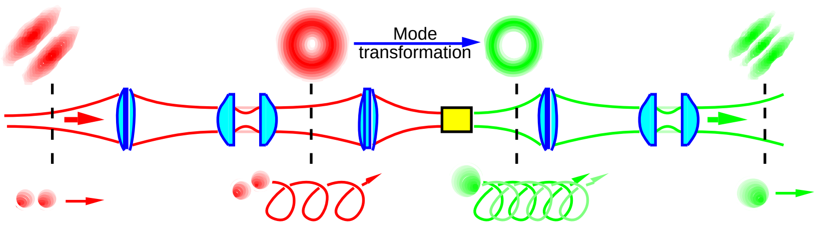
[5] N B Simpson, D McGloin, K Dholakia, L Allen and M J Padgett, Submitted to J. Mod. Opt. (August 1997)

Typically the trapped particle rotates at a frequency of several Hertz and although the required torque could, in principle, be calculated from the viscosity of the fluid and related to the predicted $\ell\hbar$ per photon, the errors associated with this were likely to render any result only approximate. A far better method is to compare the torques arising from the transfer of orbital angular momentum to a torque produced by circularly polarised light which is already well known. We demonstrated the perfect cancellation of the spin and orbital terms which confirms that the orbital angular momentum is indeed $\ell\hbar$ per photon[6]. The figure shows successive frames of video footage taken using the St Andrews optical spanner. Note that the rotation of the trapped particle can be stopped exactly by polarising the light such that the spin component of the angular momentum is subtracted from the orbital angular momentum. This work has been widely recognised in the scientific community as being of both fundamental interest and as having many possible applications in areas such as biotechnology and micromechanics. The work has been reported by *Science* covered by the BBC Radio 4 programme “Science Now”, reported by the Financial Times and featured by the Big Issue!.



The role of orbital angular momentum in nonlinear optics

In keeping with the “blue-sky” nature of this grant scheme, we completed preliminary studies on the frequency doubling of *LG* laser modes. The helical wave fronts associated with *LG* modes imply that the Poynting vector follows a spiral path as it propagates[7]. In isotropic media the wave vector follow the same path as the Poynting vector. Consequently, these findings led us to consider the behaviour of *LG* modes within nonlinear optical interactions. When a *LG* mode is frequency doubled, the we found that a mode transformation occurs, doubling the azimuthal index of the mode.



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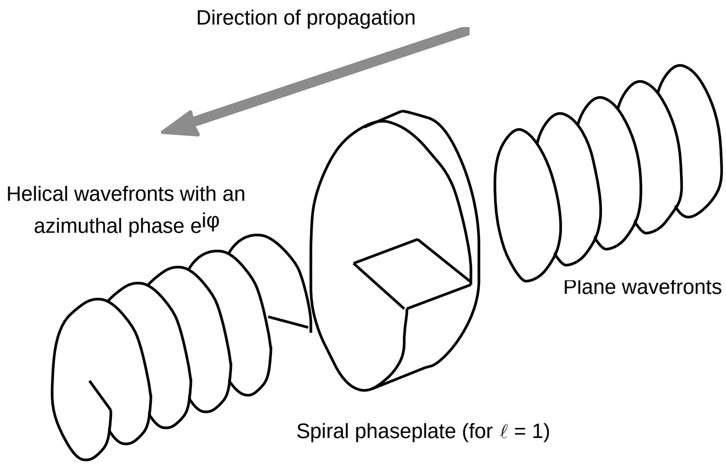
[6]N B Simpson, K Dholakia, L Allen and M J Padgett, Opt. Lett. **22**, 52-54 (1997)

[7]M J Padgett and L Allen, Opt. Commun. **121,** 36, (1995)

For example. an input $p = 0$, $\ell = 1$ mode transforms to a $p = 0$, $\ell = 2$ mode (see dia). We have explained this observation, both in terms of the phase-matching and in terms of the conservation of orbital angular momentum within the light beam [8,9]. This work acted as a pump primer for a new research area which has now being funded by the EPSRC.

The first observation of the rotational frequency shift

Within the "blue-sky" nature of this grant scheme we extended the generation of Laguerre-Gaussian modes to the mm-wave region of the spectrum. This is particularly interesting as the orbital angular momentum of $\ell\hbar$ per photon results in more angular momentum per unit power as the wavelength gets longer. Our mm-wave mode converter consists of a Teflon disc the thickness of which increases with azimuthal angle, giving a radial step in the surface. The step height is an integer multiple of wavelengths such that a transmitted Hermite-Gaussian beam has a helical phase structure superimposed upon it, giving a good approximation to Laguerre-Gaussian beam[10].

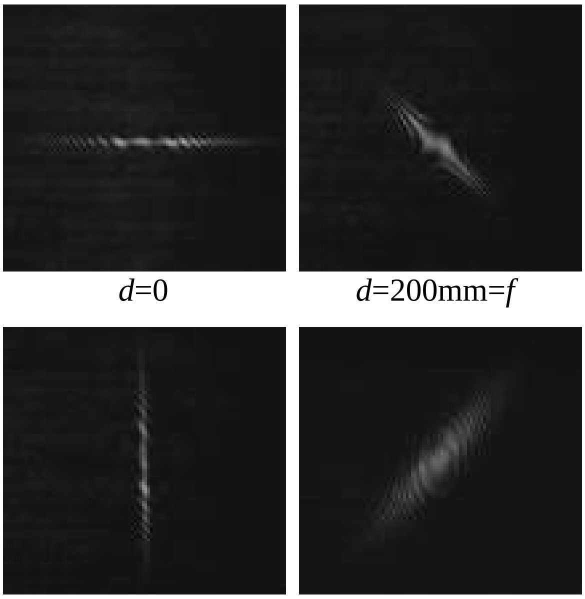


We have now used these mm-wave beams to demonstrate the rotational frequency shift recently predicted by Bialynicki-Birula and Bialynicka-Birula[11]. We use a rotating Dove prism to rotate the beam as seen by the detector. By configuring the experiment as an interferometer we detect the difference frequency between the rotating and none rotating beams by combining them onto a point detector. For a source rotating at a frequency Ω we confirmed that the observed frequency shift is given by $\ell\Omega$ [12]. We also link these observations to earlier work on the angular Doppler shift[13].

Generation of beams with very high orbital angular momentum

[8]K Dholakia, N B Simpson, L Allen and M J Padgett, **Post deadline** paper, **QELS'96**, QPD6 (1996)
[9]K Dholakia, N B Simpson, M J Padgett and L Allen, Phys. Rev. A, **54**, R3742-R3745 (1996)
[10] G A Turnbull, D A Robertson, G M Smith, L Allen and M J Padgett, Opt. Commun. **127** 183-188 (1996)
[11]I Bialynicki-Birula and Z Bialynicka-Birula, Phys. Rev. Lett. **78**. 2539 (1997).
[12]J Courtial, K Dholakia, L Allen and M J Padgett,Submitted to Phys. Rev. Lett. (November 1997)
[13]B A Garetz, J. Opt. Soc. Am. **71**, 609 (1981)

All beams with an azimuthal phase term, $e^{i\ell\phi}$, of which Laguerre-Gaussian beams are an example, have an orbital angular momentum of $\ell\hbar$ per photon. The mode index of a Laguerre-Gaussian can assume any integer value, but in practical systems it is likely to be low. Within this grant we have show how a cylindrical lens system can be used to produce light beams with an arbitrary orbital angular momentum as high as 10,000$\hbar$ per photon[14] (see opposite). The orbital angular momentum per photon is given by



d=0 *d*=200mm=*f* *d*=400mm=2*f* *d*=∞

$$\delta L_z = \frac{k}{f}\frac{\left(w_x^2 - w_y^2\right)}{4}\sin 2\alpha \times \hbar$$

For highly elliptical beams a few millimetres in size, at optical wavelengths, this can easily exceed 10,000$\hbar$ per photon. Only the f-number and aperture of the cylindrical lens limits the amount of orbital angular momentum that can be transferred. It is worth noting that, unlike the Laguerre-Gaussian beams, the elliptical beams have an on-axis intensity.

Training arising directly and impart from this grant

Neil Simpson (PhD student 1995-97). Neil was the research student chiefly dedicated to the development of the optical spanner. His work led to a number of high quality publications and presentations at international conferences. Immediately after finishing his PhD he joined British Aerospace working as part of the new Airbus development team.

Johannes Courtial (PhD student 1996-). Johannes is the research student who has opened up many of the new research areas pioneered under this grant. Primarily responsible for the identification of a class of astigmatic beams with extremely high orbital angular momentum and the extension of the work to the observation of the rotational frequency shift he also has a number of high quality publications and presentations at international conferences.

Kishan Dholakia (Research assistant 1996-1997). On joining the group at the beginning of this grant Kishan was chiefly responsible for the work on the frequency doubling of Laguerre-Gaussian modes. His experience with optical systems was key also to obtaining the definitive experiments concerning the comparison between spin and orbital angular momentum within the optical spanner. At the beginning of 1997 Kishan won a research fellowship from the Royal Society of Edinburgh to apply his new found expertise in Laguerre-Gaussian modes with his pre St Andrews expertise in atom/ion

[14] J Courtial, K Dholakia, L Allen and M J Padgett, Opt. Commun. **144**, 210-213 (1997)

trapping. This unique combination of techniques seems set to open new fields in atom optics and atom guiding. Kishan has now been funded by the EPSRC to pursue this work.

Publications arising in main from this grant

The generation of free-space Laguerre-Gaussian modes at millimetre-wave frequencies by use of a spiral phase plate, G A Turnbull, D A Robertson, G M Smith, L Allen and M J Padgett Opt. Commun. **127** 183-188 (1996)

Second harmonic generation using Laguerre-Gaussian laser modes, K Dholakia, N B Simpson, M J Padgett and L Allen Phys. Rev. A, **54** R3742-R3745 (1996)

Optical Tweezers and Spanners, M J Padgett and L Allen Physics World **10** 35-38 (1997)

Gaussian beams with very high orbital angular momentum, J Courtial, K Dholakia, L Allen and M J Padgett Opt. Commun. **144**, 210-213 (1997)

The transfer of orbital angular momentum from a stressed fibre-optic waveguide to a light beam, D McGloin, N B Simpson and M J Padgett Accepted for publication in Appl. Opt. (July 1997)

Optical tweezers with increased trapping efficiency using Laguerre-Gaussian laser modes, N B Simpson, D McGloin, K Dholakia, L Allen and M J Padgett Submitted to J. Mod. Opt. (August 1997)

Measurement of the rotational frequency shift imparted to a rotating light beam possessing orbital angular momentum, J Courtial, K Dholakia, L Allen and M J Padgett Submitted to Phys. Rev. Lett. (November 1997)

Second Harmonic Generation and the Orbital Angular Momentum of Light, K Dholakia, N B Simpson, L Allen and M J Padgett **Post deadline** paper, **QELS'96**, QPD6 (1996)

The mechanical equivalence of the spin and orbital angular momentum of light: optical spanners, N B Simpson, K Dholakia, L Allen and M J Padgett **Post deadline** paper, **IQEC'96**, ThP3 (1996)

Light beams possessing large quantities of angular momentum, J Courtial, K Dholakia, L Allen and M J Padgett **QELS’97**, QThB2 (1997)

Optical tweezers with increased trapping efficiency, N B Simpson, L Allen and M J Padgett **CLEO'97**, CTuD7 (1997)

Optical tweezers and spanners, M J Padgett National Quant. Elec. Conf. **QE-13,** Invited Paper (1997)

Propagation of frequency-doubled Laguerre-Gaussian beam and the role of orbital angular momentum, J K Courtial, K Dholakia, L Allen and M J Padgett National Quant. Elec. Conf. **QE-13,** P6-1 (1997)

The transfer of orbital angular momentum to a light beam using a stressed fibre-optic waveguide, D McGloin, N B Simpson and M J Padgett National Quant. Elec. Conf. **QE-13,** P6-2 (1997)

Publications arising in part from this grant

The mechanical equivalence of the spin and orbital angular momentum of light : an optical spanner, N B Simpson, K Dholakia, L Allen and M J Padgett Opt. Lett. **22**, 52-54 (1997)

The mechanical equivalence of the spin and orbital angular momentum of light: optical spanners, N B Simpson, K Dholakia, L Allen and M J Padgett **Post deadline** paper, **IQEC'96**, ThP3 (1996)

Second harmonic generation and the orbital angular momentum of light, K Dholakia, N B Simpson, L Allen and M J Padgett **Post deadline** paper, **QELS'96**, QPD6 (1996)